Optimal Frequency Response Modification by Added Passive Structures

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Abstract

TWO frequency response optimization methods for vibrating structures are developed for appending absorbers to the system. The methods are suitable for discrete models with a large number of degrees of freedom and are applied to obtain optimal broadband response. Reanalysis and modal synthesis techniques are used in the structural dynamic analysis phase of the design algorithm and optimization is carried out by a feasible directions approach. These optimal design algorithms have been applied to a 39 degree-of-freedom helicopter model with discrete conventional absorbers and beam absorbers.

Contents

A Substructuring Method for Optimal Appendant System Design

This method is applied when the main system remains unchanged while the property matrices for the appended absorber systems contain all of the design variables. It is then natural to treat the component systems separately when computing the response of the compound structure within the optimal design algorithm. This substructuring approach to the design of linear appendant systems, developed as a reanalysis procedure, allows sufficient reduction in computing time so that systems containing large numbers of degrees of freedom may be treated.[1]

This optimal design algorithm can be stated as follows:

1) Assemble the initial system property matrices M_0, C_0, and K_0 for mass, damping, and stiffness, respectively.

2) Discretize the excitation frequency range $[\omega_a, \omega_b]$ into a set of L frequencies

$$\omega_{k+1} = \omega_k + \Delta\omega_k$$

for

$$1 \leq k \leq L-1, \qquad \omega_1 = \omega_a, \qquad \omega_L = \omega_b$$

3) For each frequency ω_k, compute and store the response corresponding to the initial property matrices M_0, C_0, and K_0.

4) For each frequency ω_k, compute and store the elements of the system receptance matrix needed in the reanalysis algorithm.

5) Guess initial values for the optimum design variables of the absorber and set the nonlinear programming iteration count equal to 1.

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6) Apply iteration k of the optimization algorithm as follows:

a) Compute the necessary components of the response by a reanalysis algorithm.[1]

b) Compute response derivatives by finite differences.

c) Using steps a and b, compute the value of the chosen cost function.

d) Compute the gradient of the cost function using step b or a finite difference formula.

e) Determine a new vector of design variables from the feasible directions algorithm.

7) Check the termination criteria:

a) Has the cost function changed during the last n iterations?

b) Has the number of iterations k reached its maximum allowable value?

8) Stop if at least one of the termination criteria is satisfied; otherwise, increase k and go to step 6.

A Modal Synthesis Method for Optimal Appendant System Design

The exact reanalysis approach of the previous method is replaced here with an approximate one based on modal synthesis. The basic idea in modal synthesis is to find the eigenvalues and eigenvectors of a system in terms of the modal properties of its components. Hurty[2] introduced the concept in 1960 and many variations have appeared in the literature since then.[3-6] Although the method has been extended to complex modes,[7,8] it is usually applied to undamped modes. Here, a two-step method is used to avoid the complications of the techniques presented in Ref. 7:

1) Use component mode synthesis to compute the undamped modal properties of the composite system (main system with designed appendant systems attached to it). These modes are to be used as the assumed modes in the next step.

2) Introduce damping as a modification using an assumed modes approach to obtain the complex modal properties of the composite system.

The accuracy of this method depends on the number of component modes retained in the modal synthesis (step 1) and the number of system modes n_M used in the assumed modes analysis (step 2). The order of the condensed eigenvalue problem for the damped system is $2n_M$, independent of the number of interface degrees of freedom. However, this modal approach involves more programming and more calculation than the substructuring approach of the previous section. In return for this increase in computational complexity, the modal method gives the response as an explicit function of excitation frequency, thus eliminating the need to solve linear equations for each frequency. This method becomes efficient when the number of modes retained in the modal synthesis and in the assumed modes analysis can be reasonably small without an unacceptable loss in accuracy and when it is necessary to evaluate response over a broad frequency band.

The optimal design proceeds as described in the previous section except that the reanalysis method is replaced by modal synthesis in the repetitive response analyses within the feasible directions algorithm.

Conclusions

The methods described have been applied successfully to two structural models. The first optimal design method has been applied to the problem of designing three conventional vibration absorbers for a 39 degree-of-freedom undamped helicopter model made up of 14 beam elements and 2 discrete springs. The design goal was to suppress the first three resonances at the pilot's seat when a couple was applied at the rotor hub. The second method has been used to design a beam absorber with structural damping for the same helicopter model with the same design goal.

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